27 Anomaly Mediation Part II

27.1 The μ problem

Recall that in order to obtain a viable mass spectrum in the MSSM, we needed μ and b terms

$$W = \mu H_u H_d \tag{27.1}$$
$$V = b H_u H_d \tag{27.2}$$

with

$$b \sim \mu^2 \tag{27.3}$$

In anomaly mediated models we need

$$\mu \sim \frac{\alpha}{4\pi}\frac{F}{M} \tag{27.4}$$

If we introduce a coupling to the SUSY breaking field

$$W = \mu \frac{\Sigma}{M_{\rm Pl}^3} H_u H_d \tag{27.5}$$
$$\tag{27.6}$$

we get

$$b \quad = \quad 3\frac{F_\Sigma}{M_{\rm Pl}}\mu \sim \frac{12\pi}{\alpha}\mu^2 \tag{27.7}$$

A more complicated possibility is

$$\mathcal{L} = \alpha \int d^4\theta \frac{X + X^\dagger}{M_{\rm Pl}} H_u H_d \frac{\Sigma\Sigma^\dagger}{M_{\rm Pl}^2} + h.c. \tag{27.8}$$

After rescaling

$$\frac{\Sigma H_i}{M_{\rm Pl}} \to H_i \ . \tag{27.9}$$

we have

$$\mathcal{L}_{\rm eff} = \int d^4\theta \frac{X + X^\dagger}{M_{\rm Pl}} H_u H_d \frac{\Sigma^\dagger}{\Sigma} + h.c. \tag{27.10}$$

and

$$\mu = \alpha \left(\frac{F_X^\dagger}{M_{\rm Pl}} + \frac{F_\Sigma^\dagger}{M_{\rm Pl}} \right) \tag{27.11}$$

$$b = \alpha \left(\frac{F_X}{M_{\rm Pl}} \frac{F_\Sigma^\dagger}{M_{\rm Pl}} - \frac{F_X^\dagger}{M_{\rm Pl}} \frac{F_\Sigma}{M_{\rm Pl}} \right) \tag{27.12}$$

which vanishes if $F_\Sigma \propto F_X$. At one loop a b term is generated. To canonically normalize the Higgs fields we rescale:

$$H_i' = Z_i^{1/2} \left(1 - \frac{1}{2} \gamma_i \frac{F_\Sigma}{M_{\rm Pl}} \theta^2 \right) |_{\Sigma = M_{\rm Pl}} H_i \ . \tag{27.13}$$

Then we find:

$$\begin{aligned} b &= \alpha \frac{F_X^\dagger}{2 M_{\rm Pl}} \left(\gamma_u \frac{F_\Sigma}{M_{\rm Pl}} + \gamma_d \frac{F_\Sigma}{M_{\rm Pl}} \right) \\ &= \mathcal{O}(\mu^2) \end{aligned} \tag{27.14}$$

We can generate the required interaction with a 5D toy model, where the fifth dimension has radius r_c. Recall that for $r < r_c$ the gravitational potential is

$$\frac{1}{r^2 M_5^3} \tag{27.15}$$

rather than

$$\frac{1}{r M_{\rm Pl}^2} \ . \tag{27.16}$$

Matching at $r = r_c$ we have

$$M_{\rm Pl}^2 = r_c M_5^3 \tag{27.17}$$

We introduce a massive vector field V which propagates in the 5D bulk (recall that it has canonical dimension 3/2). Integrating over the fifth dimension we assume the 4D effective action has the form:

$$\mathcal{L} = \int d^4\theta \left(r_c m^2 V^2 + aV(x + X^\dagger) M_5^{3/2} + \frac{bV}{M_5^{3/2}} H_u H_d \right) \frac{\Sigma \Sigma^\dagger}{M_{\rm Pl}^2} + h.c. \tag{27.18}$$

Integrating out V gives

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \frac{ab}{r_c m^2}(X + X^\dagger) H_u H_d \frac{\Sigma\Sigma^\dagger}{M_{\text{Pl}}^2} + h.c. \tag{27.19}$$

with

$$\begin{aligned} r_c M &\sim \mathcal{O}(1) \\ m &\sim \mathcal{O}(M_{\text{Pl}}) \\ ab &\sim \mathcal{O}(\alpha) \end{aligned} \tag{27.20}$$

we arrive at the required interaction.

27.2 Slepton masses

Recall that the quark and slepton masses had the form:

$$M_q^2 = \frac{1}{2}\left[\frac{C_2(r)}{4\pi^2} b g^4 + a\lambda^2(e\lambda^2 - f g^2)\right] \frac{|F_\Sigma|^2}{M_{\text{Pl}}^2} \ . \tag{27.21}$$

and b is negative for $SU(2)_L$ and $U(1)_Y$, so the sleptons are tachyonic. There are several ways to fix this problem:

- introduce new Higgs fields with large Yukawa couplings
- introduce new asymptotically free gauge interactions for sleptons, this requires that the leptons and sleptons are composite
- introduce new bulk fields which couple leptons and the DSB fields on the other brane
- introduce a heavy threshold (like messengers) with a light singlet.

We will only consider the last possibility, which is also the simplest. This scenario is known as “Anti-Gauge Mediation”.

We consider a model with a singlet X and N messengers ψ and $\overline{\psi}$ in $\Box$'s and $\overline{\Box}$'s of $SU(5)$ with a superpotential

$$W = \lambda X \psi \overline{\psi} \tag{27.22}$$

X is pseudo-flat, it gets a mass through anomaly mediation

$$\begin{aligned} V(X) &= m_X^2(X)|X|^2 \\ &= \frac{N}{16\pi^2}\lambda^2(X)\left[A\lambda^2(X) - Cg^2(X)\right]\frac{|F_\Sigma|^2}{M_{\text{Pl}}^2}|X|^2 \ . \end{aligned} \tag{27.23}$$

If the messengers have some asymptotically free gauge interactions (embedded in $SU(N)$) then it is possible to arrange the parameters so the $m_X^2(X)$ changes sign, and X is stabilized nearby (this is the Coleman-Weinberg mechanism again). Take

$$\langle X \rangle = M \tag{27.24}$$

then

$$\frac{F_X}{\langle X \rangle} \sim \frac{\lambda}{16\pi^2} \frac{F_\Sigma}{M_{\rm Pl}} \tag{27.25}$$

The couplings of the low energy theory depend on

$$\widetilde{X} = X \frac{M_{\rm Pl}}{\Sigma} \tag{27.26}$$

so for $M \ll M_{\rm Pl}$

$$\frac{F_{\widetilde{X}}}{\langle \widetilde{X} \rangle} = \frac{F_X}{M} - \frac{F_\Sigma}{M_{\rm Pl}} \approx -\frac{F_\Sigma}{M_{\rm Pl}} \tag{27.27}$$

So Taylor expanding the coefficient of $W^\alpha W_\alpha$ in superspace we find a gaugino mass:

$$\begin{aligned} M_\lambda &= -\frac{1}{2\tau} \frac{\partial \tau}{\partial \ln \Sigma}\Big|_{\Sigma = M_{\rm Pl}} \frac{F_\Sigma}{M_{\rm Pl}} \\ &= \frac{1}{2\tau}\left(\frac{\partial \tau}{\partial \ln \mu} + \frac{\partial \tau}{\partial \ln X} \right) \frac{F_\Sigma}{M_{\rm Pl}} \\ &= \frac{\alpha(\mu)}{4\pi}(b - N)\frac{F_\Sigma}{M_{\rm Pl}} \end{aligned} \tag{27.28}$$

We can also Taylor expand the matter wavefunction renormalizations in superspace to find a squark or slepton mass squared:

$$\begin{aligned} M_q^2 &= -\left(\frac{\partial}{\partial \ln \mu} + \frac{\partial}{\partial \ln |X|} \right)^2 \ln Z(\mu, |X|) \frac{|F_\Sigma|^2}{4M_{\rm Pl}^2} \qquad (27.29) \\ &= \frac{2C_2(r)b}{(4\pi)^2}\left[\alpha^2(\mu) - \alpha^2(\mu)\frac{N}{b} + (\alpha^2(\mu) - \alpha^2(M))\left(\frac{N^2}{b^2} - \frac{N}{b} \right) \right] \frac{|F_\Sigma|^2}{M_{\rm Pl}^2} \end{aligned}$$

The first term in the gaugino mass and squark/slepton mass squared formulas is just the anomaly mediation term, the second term is the gauge mediation term but with the opposite sign. This is the reason the these models are called anti-gauge mediation models. The final term in the squark/slepton mass squared formula is just the RG running with the gaugino mass, which we have seen many times. For large enough N this term can dominate for the sleptons. We cannot take M too large or our approximations break down and higher dimension operators can start to dominate. For example

$$\int d^4\theta \frac{X^\dagger X}{M_{\rm Pl}^2} Q^\dagger e^V Q \tag{27.30}$$

would give a mass squared

$$M_q^2 = -\frac{|F_X|^2}{M_{\rm Pl}^2} \tag{27.31}$$

For M close to $M_{\rm GUT}$ we need $N \geq 4$.

With the addition of another singlet field S this model can also generate μ and b terms. Take the superpotential to be

$$\int d^2\theta \lambda S H_u H_d + \frac{k}{3} S^3 + \frac{y}{2} S^2 X \tag{27.32}$$

At one loop a kinetic mixing develops:

$$\int d^4\theta \widetilde{Z} S X^\dagger + h.c. \tag{27.33}$$

For $\langle X \rangle \neq 0$, S is massive an can be integrated out:

$$S \sim -\frac{\lambda}{y} \frac{H_u H_d}{X} \tag{27.34}$$

This generates the interaction

$$\mathcal{L}_{\rm eff} = \int d^4\theta \frac{X^\dagger}{X} H_u H_d \widetilde{Z}\left(\frac{|X| M_{\rm Pl}}{\Lambda |\Sigma|}\right) + h.c. \tag{27.35}$$

generates a μ term at one loop:

$$\mu = \frac{1}{2} \frac{\partial \widetilde{Z}}{\partial \ln |X|} \tag{27.36}$$

and a b term at two loops:

$$b = \frac{1}{4} \frac{\partial^2 \widetilde{Z}}{\partial (\ln |X|)^2} \tag{27.37}$$

27.3 Gaugino Mediation

Since RG generates positive mass squared for all the scalars it is possible to consider models where to leading order only the gaugino gets a mass. A simple way to set this up to to have a compact dimension with a radius around

$$r_c \sim \frac{1}{M_{\rm GUT}} \tag{27.38}$$

and let the gauge fields of the MSSM propagate in this extra dimension, with the source of SUSY breaking being another brane at the other end of the fifth dimension. The 4D gauge coupling is related to the 5D coupling by

$$g_4^2 = \frac{g_5^2}{r_c} \tag{27.39}$$

Since there is no chirality in 5D, the minimal SUSY theory has $\mathcal{N} = 2$. The 5D $\mathcal{N} = 2$ gauge supermultiplet breaks into a 4D gauge supermultiplet and and adjoint chiral multiplet:

$$(A_N, \lambda_L, \lambda_R, \phi) \to (A_\nu, \lambda_L) + (\phi + iA_5, \lambda_R) \tag{27.40}$$

The compactification can be chosen so that the chiral multiplet vanishes on our brane. SUSY breaking on the other brane can be communicated to the gauge fields by

$$\begin{aligned} \mathcal{L} &\propto \int dx_5 \int d^2\theta \left(1 + \delta(x_5 - r_c)\frac{X}{M^2}\right) W^\alpha W_\alpha + h.c. \\ &\propto r_c \lambda^\dagger \overline{\sigma}^\mu D_\mu \lambda + \frac{F_X}{M^2}\lambda^\dagger\lambda + \ldots \end{aligned} \tag{27.41}$$

so

$$M_\lambda = \frac{1}{r_c M}\frac{F_X}{M} \tag{27.42}$$

Bulk gluino exchange gives a squark mass:

$$M_Q^2 \sim \frac{g_5^2}{16\pi^2}\left(\frac{F_X}{M^2}\right)^2 \frac{1}{r_c^3} = \frac{g_4^2}{16\pi^2} M_\lambda^2 \tag{27.43}$$

which is suppressed relative to the gluino mass squares, so for $r_c \ll M_{\rm weak}^{-1}$, the 4D RG running dominates.

References

[1] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," hep-th/9810155.

[2] A. Pomarol and R. Rattazzi, "Sparticle masses from the superconformal anomaly," JHEP **05** (1999) 013 hep-ph/9903448.

[3] Z. Chacko, M.A. Luty, I. Maksymyk and E. Ponton, "Realistic anomaly-mediated supersymmetry breaking," hep-ph/9905390.

[4] D.E. Kaplan, G.D. Kribs and M. Schmaltz, "Supersymmetry breaking through transparent extra dimensions," hep-ph/9911293.

[5] Z. Chacko, M.A. Luty, A.E. Nelson and E. Ponton, "Gaugino mediated supersymmetry breaking," hep-ph/9911323.